A Novel Approach to Determine Wet Restitution Coefficients Through a Unified Correlation and Energy Analysis

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Wet particle interactions are observed in many applications, for example, pharmaceutical, food, agricultural, polymerization, agglomeration, and coating, in which an accurate evaluation of the wet restitution coefficient (e_{wet}) is crucial to understand the particle flowability, operating conditions and product size distribution. Experiments were performed to measure the wet restitution coefficient by impacting a spherical particle on a stationary plate covered with a thin liquid layer of water or glycerol solution in this work. Furthermore, novel approaches for estimation of e_{wet} were developed using dimensional analysis (using the Buckingham π theorem and regression analysis) in combination with energy budget analysis. In the correlation development, the dominant physical properties of solid and liquid, particle impact velocity and liquid layer thickness are grouped into well-known dimensionless numbers viz. Reynolds, Weber and Stokes. Whereas in the energy analysis, the energy dissipation rates were determined for five distinct collision phases, that is, dipping, dry collision, undipping, formation and breakage of the liquid bridge, and added mass. The efficacy of the developed approaches was analyzed by comparing obtained results with experiments and an elastohydrodynamic model, and a modified elastohydrodynamic model. © 2014 American Institute of Chemical Engineers *AIChE J*, 61: 769–779, 2015
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Introduction

There exist numerous applications in chemical, pharmaceutical, food, agricultural, polymerization, agglomeration, coating industries, and so forth involving handling, processing, and transport of particles. The particulate flow characteristics in these systems highly depend on the effective particle interactions or impacts. A small change in these interactions owing to the chemical and/or physical transformations significantly alter the particle morphology and hence overall system dynamics. For instance, during agglomeration and coating, a liquid binder mixed with a solvent is often atomized on the dry fluidizing particles to produce a coated powder, agglomerates or granules. The injection of the liquid may result in partial or complete particle wetting. The presence of liquid on the particle surface dramatically modifies the energy dissipation rate and hence particle interactions and resulting particle size distributions.[1] Therefore, a detailed study to estimate the wet interactions via an effective (wet) restitution coefficient has rapidly gained attention to improve the product quality, system efficiency, and production rate.

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Over the past decade, extensive experimental research has been carried to study the restitution coefficient under dry conditions (e_{dry}), defined as the ratio of the particle impact velocities after and before collision. Generally, the restitution coefficient provides information about kinetic energy dissipated during the collision; $e_{dry} = 1$ indicates a perfectly elastic collision, conserving the total kinetic energy associated with the particle and $e_{dry} = 0$ indicates a perfectly inelastic collision where the total kinetic energy is dissipated in the collision and the particle will not rebound. For an inelastic dry collision, the restitution coefficient is often lower than 1, that is, the energy associated with the particles is dissipated into various effects like vibrations, sound, and plastic or viscoelastic deformation of the materials. The energy dissipation rate not only depends on the material properties but also on the relative particle velocity. Representative studies highlighting dry collisions include: photographic study of particle impacts and effect of the impact velocity on the restitution coefficient of various martials[2]; determination of the restitution coefficient and sticking velocities of a glass particle on a flat plate covered with a dry silica powder layer[3]; measurement of the tangential restitution coefficient ($e_{dry,t}$) by impacting an aluminum oxide particle on soda lime glass and aluminum alloy plates.[4] Mueller et al.[5] extended the impact study for an oblique impact, with impact angles ($\theta_i = 0$–$90°$) by estimating the dry normal and tangential restitution coefficient. They revealed that at a constant impact velocity, the normal restitution coefficient approximately remains constant with impact angle while the tangential restitution coefficient significantly decreases up to 20–30° and then increases reaching a maximum value of 1 at 90°. This may be due to a change in the particle collision behavior (pure slip, partial slip and sliding, and pure sliding) with an impact angle. At a high impact angle, a negligible amount of energy is dissipated during particle impact, hence a high tangential restitution coefficient is obtained. Mangwandi et al.[6] determined the restitution coefficient for melt, wet, and binder-less granules. Recently, dry experiments were performed by Marinack et al.[7] to determine the restitution coefficient by analyzing the effect of the particle size, material properties, and particle impact velocities.

However, only a limited number of experimental studies have been reported for the estimation of the restitution coefficient in the presence of a liquid (i.e., wet restitution coefficient, e_{wet}) and even fewer studies on the development of a correlation and/or a phenomenological model. Joseph et al.[8] evaluated the wet restitution coefficient by means of high speed imaging of the particle rebound ($d_p = 3$–5 mm) from a glass wall covered with glycerol and water using a pendulum string. Their investigations suggest that the particle rebound velocity mainly depends on Stokes number and weakly on the material properties (Young's modulus and Poisson's ratio). Further, at low impact velocities and thin liquid layer surface roughness plays an important role. Davis et al.[9] calculated the wet restitution coefficient by dropping spherical particles (Nylon 66 and stainless steel of $d_p = 3.2$–6.4 mm) on a smooth quartz plate covered with a thin liquid layer (δ_l) of silicon-based oils in the range of $\delta_l = 80$, 150, and 250 (μm). Additionally, an approximate model based on lubrication theory for undeformed particles and scaling relations for elastic deformation and rebound was proposed, and model results were compared with experimental findings. Gondret et al.[10] quantified the wet restitution coefficient of various particles (d_p=1–6 mm) on a plate covered with various liquids of varying viscosities. In addition, rebound trajectories of the particles were calculated by solving the equation of motion considering external forces such as gravity, drag and added-mass forces. Fu et al.[11,12] correlated properties of the granules (in terms of binder content, porosity, shape, and surface condition) prepared under different operating conditions by estimating the coefficient of restitution. Kantak and Davis[13] experimentally studied the collisions of the particles on a solid surface covered with a thin layer of wet and dry fabric. Their results showed that, the dry restitution coefficient slightly decreases with impact velocity due to larger inelastic losses in residual deformation. Note that, the dry restitution coefficient is much lower in the presence of a fabric, due to the higher inelastic losses associated with the fabric. The wet restitution coefficient also slightly decreases for investigated particle impact velocities. This may be due to the sufficient required energy to overcome the viscous losses that are stored in solid deformation. Stocchino and Guala[14] estimated the wet restitution coefficient for steel particles on a plate with water and carboxymethyl cellulose using a digital image analysis technique. Antonyuk et al.[15] obtained experimental results for the particle collision (at 2.36 m/s impact velocity) and compared those with numerical results obtained by numerically solving the equation of motion by considering various forces acting on the particles. Mueller et al.[5] evaluated the normal restitution coefficient ($e_{wet,n}$) for three wet granules (Al_2O_3, zeolite 4A and sodium benzoate) on a hardened steel plate. Subbarao,[16] performed experiments by impacting a brass particle on a liquid covered substrate and compared their results with an elastohydrodynamic model proposed by Ref. 9, and numerically solving the particle motion by considering various forces as reported by Ref. 15. Their results reveal revealed that the approach proposed by Antonyuk et al.[15] possesses a limited applicability at higher Stokes number compared with Davis et al.[9] Wet particle interactions were also studied by impacting two wet particles held by pendulum strings by Donahue et al.[17,18] Although their investigations are very significant, the particle collisions are often influenced by the centrifugal force and it is difficult to maintain a uniform liquid layer thickness around the particles.

The objective of this study is to develop a unified correlation for the wet restitution coefficient using Buckingham π theorem and quantifying the energy dissipation during five distinct collision phases. In the correlation development, the dominant physical properties of solid and liquid, particle impact velocity, and liquid layer thickness are grouped into well-known dimensionless numbers viz. Reynolds, Weber, and Stokes. The efficacy of the developed approaches were analyzed by comparing the obtained results with an elastohydrodynamic model[9] and experimental results obtained by impacting a particle on a stationary plate covered with a thin liquid layer of water and glycerol. In this work, investigations were performed at a room temperature and atmospheric pressure without considerable liquid evaporation. Also the effect of particle roughness and angular velocity were neglected.

We realize that the system studied, in this work, is a simplification of the complex particle–particle interactions that occur during fluidization. However, experiments under actual fluidizing condition are virtually impossible to perform and to control. Therefore, this work should be viewed as a first step to determine and verify the wet restitution coefficient used for a wet binary collision. As the collision time is usually very short, knowledge of the net energy dissipation during the wet collisions through various forces (gravitational, surface tension, buoyancy, drag, contact between the particle and the wall, viscous and capillary) is more important than the actual collision behavior. So, the expressions for the wet restitution coefficient can be used as input for large scale models (computational fluid dynamics-discrete element model [CFD-DEM]) that can be used to study the variation in the flow patterns under wet conditions in, for example, fluidized bed granulator study. Additionally this study will also be useful to validate the direct numerical simulations of the complex multiphase flows used to study wet–particle interactions, for example, see Jain et al.[19] The organization of this work is as follows: first, the experimental analysis will be reported, followed by dimensional analysis and energy analysis. Then, obtained results will be compared with the experiments.

Experimental Setup

In this study, the experiments were performed in a similar way as described by Antonyuk et al.,[15] a schematic of the experimental setup is shown in Figure 1. During experiments, a spherical glass particle (of $d_p = 2.5 \pm 0.02$ mm and $\rho_p = 2526$ kg/m^3; obtained from Sigmund Lindner GmbH,

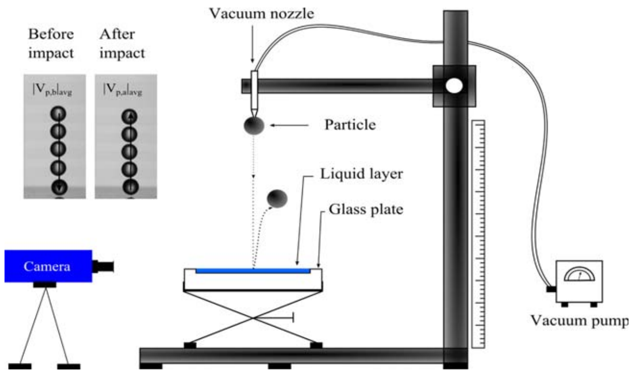


Figure 1. Schematic of the experimental setup for estimation of the dry and wet restitution coefficient on a stationary glass plate.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Germany) was impinged on a stationary glass plate (soda lime) of dimension $W \times D \times H = 8 \times 8 \times 2$ cm^3, covered with a thin liquid layer. SEM snapshots of the used particle displaying surface morphology are shown in Figure 2. It should be noted that, collisional energy dissipation also depends on the plate thickness. The experiments were performed on a flat glass plate (of the same material as that of particle) with a thickness higher than five times the particle diameter. The experiments were performed by changing the particle impact velocity ($v_{p,0} = 0.9$–5 m/s) through adjustment of the vacuum nozzle position, the liquid viscosity, density, surface tension, and the liquid layer thickness ($l = 0$–1000 μm). Note that, the experiments were performed in a closed box to avoid an influence of the ambience on the particle deflection and to minimize the liquid evaporation rate by maintaining a constant temperature. The uniform liquid layer thickness was ensured by applying a very thin hydrophilic coating by exposing the glass plate to Ozone for 20 min. prior to each set of experiment. The particle was dropped by releasing the vacuum from the nozzle and the impact velocities were obtained by capturing high speed images using a camera (LaVision High Speed Star with a Sigma 105 F2.8 DG lens, at 4000 Hz). Other settings, such as the aperture area and the exposure time were altered to obtain clear images.

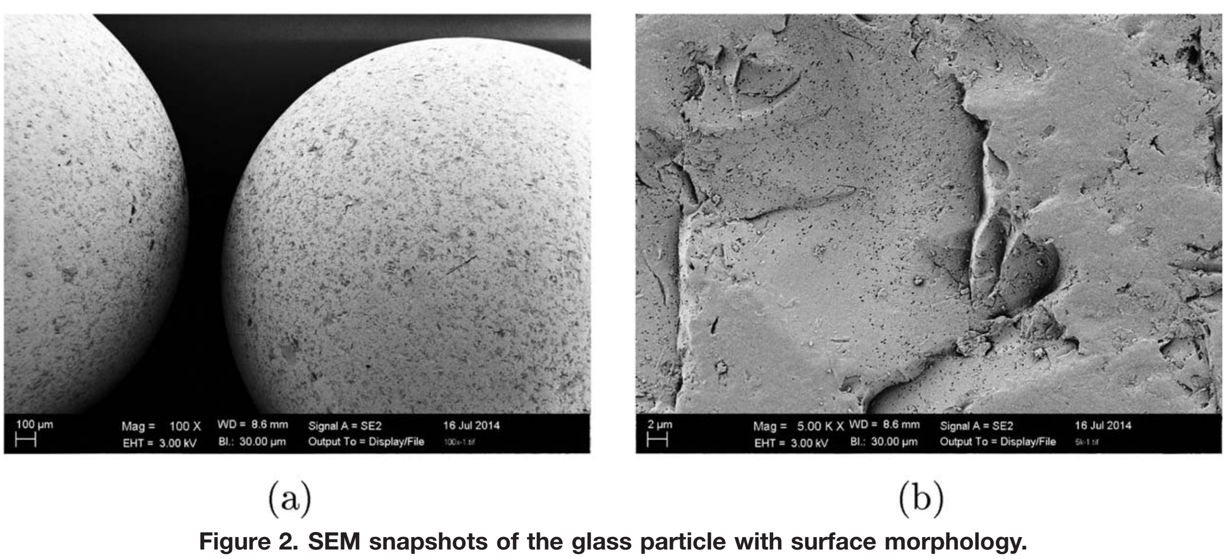
The captured images were analyzed in MATLAB (R2012 a) script by determining the particle centroid displacement after and before the collision with time and wet restitution coefficient was evaluated using Eq. 1. The pixel displacement in the images was rescaled to mm by calculating number of pixels associated with the particle size

$$e_{\text{wet}} = \frac{|v_{\text{p,a}}|_{\text{avg}}}{|v_{\text{p,b}}|_{\text{avg}}} \quad (1)$$

Each measurement was repeated 10 times to reduce the manual error, and average values of the wet restitution coefficient were calculated with standard deviation. Additionally, the particle velocity was determined by averaging 10 data points after and before impact (Figure 1 shows only five data points due to space limitations). Noted that the ratio of the particle height before and after impact can also provide meaningful information about the restitution coefficient. However, these measurements may be influenced by particle trajectories and interstitial air. The liquid layer thickness on the plate was determined by measuring total volume of the liquid layer (weight/density) and dividing it by wetted base area of the plate (5×5 cm^2). The weighing procedure was repeated after each measurement to minimize the evaporation effects.

Dimensional Analysis

Published wet collision experiments imply that the additional energy dissipated (next to the energy dissipated in a dry collision) in presence of a liquid layer depends on the particle impact velocity, physical properties of the solid and liquid, and also on the liquid layer thickness. The variables effecting the wet restitution coefficient are summarized in Table 1. For a meaningful classification of these parameters, it is required to combine them in terms of dimensionless groups characterizing ratios of the inertia, viscous, surface tension, and capillary forces. In this section, a parametric dependency of the restitution coefficient is explored by expressing it as a function of the dominant variables. The



(a) (b)

Figure 2. SEM snapshots of the glass particle with surface morphology.

Table 1. The Essential Parameters Considered During Dimensional Analysis of the Wet Coefficient Restitution

Parameter	Symbol	Dimension	MLT
Particle diameter	d_p	m	$M^0 L^1 T^0$
Density of the particles	ρ_p	kg/m^3	$M^1 L^{-3} T^0$
Particle impact velocity	$v_{p,0}$	m/s	$M^0 L^1 T^{-1}$
Liquid layer thickness	δ_l	m	$M^0 L^1 T^0$
Viscosity of the liquid	μ_l	Pa s	$M^1 L^{-1} T^{-1}$
Density of the liquid	ρ_l	kg/m^3	$M^1 L^{-3} T^0$
Surface tension of the liquid	γ_l	N/m	$M^1 L^0 T^{-2}$
Dimensionless energy loss	ΔE_{wet}	–	$M^0 L^0 T^0$

additional energy dissipated in presence of the liquid can be expressed as

$$e_{wet} = e_{dry}(1-\Delta E_{wet}) \tag{2}$$

where the dimensionless energy loss during a wet collision can be expressed as

$$\Delta E_{wet} = f(d_p, \rho_p, v_{p,0}, \delta_l, \mu_l, \rho_l, \gamma_l) \tag{3}$$

In this study, we consider two different selections of the dependent/reference variables. In Case 1, the particle impact velocity, liquid layer thickness, and viscosity were considered as dependent variables, whereas in Case 2 dependent variables were particle impact velocity, liquid viscosity, and density. After applying the Buckingham π theorem, we obtained

$$\Delta E_{wet} = f\left(\frac{d_p}{\delta_l}, \frac{\gamma_l}{\mu_l v_{p,0}}, \frac{\rho_l v_{p,0}\delta_l}{\mu_l}, \frac{\rho_p v_{p,0}\delta_l}{\mu_l}\right) \tag{4}$$

The final expression to estimate the restitution coefficient can be rearranged as

$$\Delta E_{wet} = a\left(\frac{d_p}{\delta_l}\right)^b We_p^c Re_l^d St^e \tag{5}$$

where the dimensionless groups are given by

$$Re_l = \frac{\text{liquid layer inertia force}}{\text{liquid layer viscous force}} = \frac{\rho_l v_{p,0}\delta_l}{\mu_l} \tag{6}$$

$$We_p = \frac{\text{particle inertia force}}{\text{liquid layer surface tension force}} = \frac{\delta_l v_{p,0}^2 d_p}{\gamma_l} \tag{7}$$

$$St = \frac{\text{particle inertia force}}{\text{viscous force in the liquid layer}} = \frac{m_p v_{p,0}}{6\pi\mu_l r_p^2} \tag{8}$$

Energy Analysis

In this section, we will develop a closure equation for the effective wet restitution coefficient by considering an energy balance during the collision inspired by Gollwitzer et al.[20] There is one fundamental difference between their and our approach. Gollwitzer et al. considered the net effect of different dissipational processes by taking the sum of the energy losses, using in each term the same (initial impact or rebound) velocity. In our view, this is not correct and we should divide the collision into a number of distinct phases, each with its own distinct mechanism of energy loss. At the end of each phase, the particle will have a different (lower) velocity, which will influence the forthcoming phases of the collision. The overall result is that in our case, the net effect of the different dissipational processes, is the product of the subsequent energy losses. The kinetic energy associated with the moving particle is partly dissipated into the liquid and in the form of viscoelastic and/or plastic deformation and surface wave propagation (converted into vibrations, sound, and heat effects). The total energy balance during the particle impact becomes

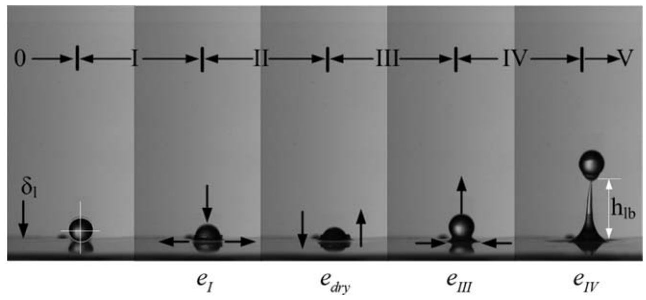


Figure 3. Schematic representation of the five collisional phases observed during the particle impact on a glass plate covered with water: I. penetration of the particle into the liquid layer; II. particle contact with the plate; III. rebound of the particle; IV. formation and breakage of the liquid bridge.

$$E_b = E_a + \Delta E_{loss} \tag{9}$$

where E_b and E_a are the total energy associated with the particle before and after the collision. Note that the energy loss (ΔE_{loss}) during the collision is not dimensionless as ΔE_{wet} in Eq. 3. ΔE_{loss} can be estimated by determining the energy dissipation rate in distinct phases observed during the particle impact on the plate covered with liquid layer as shown in Figure 3:

- Phase I: Particle penetration (i.e., dipping) into the liquid. The initial particle kinetic energy (before the penetration) is dissipated into the liquid via. viscous damping and to accelerate the displaced liquid at the location of impact.
- Phase II: Particle contact with the plate. After particle penetration into the liquid layer additional energy is dissipated associated with the particle–plate contact (i.e., solid contact). This is expected to be equal to the dry restitution coefficient.
- Phase III: Particle rebound (i.e., undipping). After particle–plate contact, the particle suddenly changes the direction by dissipating energy in viscous damping and accelerating the surrounding liquid.
- Phase IV: Formation and rupture of the liquid bridge. Just after the particle rebound, some additional energy is lost during formation and breakage of the liquid bridge.
- Phase V: Furthermore, a small amount of liquid gets attached to the particle (i.e., added mass).

The energy dissipation during the wet impact in the granular systems can be approximated to quantify the wet restitution coefficient by determining the energy losses during each phase. The particle kinetic energy at the beginning of phase i is given by

$$E_i = \frac{1}{2} m_p v_{p,i}^2 \tag{10}$$

where m_p and $v_{p,i}$, respectively, are the particle mass and the velocity at the beginning of phase i. Note that the initial velocity before impact as indicated as 0 and the final velocity is associated with phase V. Each phase is associated with its own restitution coefficient e_i, which is defined as

$$e_i = \frac{v_{p,i}}{v_{p,i-1}} = \sqrt{\left(\frac{E_{i-1} - \Delta E_i}{E_{i-1}}\right)} \tag{11}$$

Expressions for the energy dissipation ΔE_i and restitution coefficient e_i associated with each of the five phases will now be introduced.

Phase I: Ennis et al.[21] estimated a minimum particle rebound velocity by considering various forces and neglecting the capillary effects. Based on their work, the energy dissipation during viscous damping can be obtained as

$$\Delta E_{vis,I} = \frac{3}{8}\pi\mu_l d_p^2 v_{p,0} \ln\left(\frac{\delta_l}{\epsilon_{pr}}\right) - \frac{9}{128}\frac{\pi^2 \mu_l^2 d_p^4}{m_p} \tag{12}$$

where ε_{pr} is the particle roughness, which generally is in the range of a few microns. In this study, the contribution of the last term is neglected due to its very low value. The expression to estimate the energy required to accelerate the liquid after the impact is given by Ref. 20 as

$$\Delta E_{acc,I} = \frac{1}{2}\rho_l \left[\pi d_p^3 \left(\frac{\delta_l}{d_p}\right)^2 \left(\frac{1}{2} - \frac{\delta_l}{3d_p}\right)\right] \left[v_{p,0}\left(\frac{d_p}{\delta_l} - 1\right)^{0.5}\right]^2 \tag{13}$$

The restitution coefficient during Phase I can be obtained using

$$e_I = \sqrt{\left(\frac{E_0 - \Delta E_{vis,I} - \Delta E_{acc,I}}{E_0}\right)} \tag{14}$$

Phase II: The coefficient of restitution during Phase II (particle–plate contact) is approximated to be equal to that under dry conditions

$$e_{II} = \frac{v_{p,II}}{v_{p,I}} = e_{dry} \tag{15}$$

Phase III: Similar to Phase I, the viscous and particle penetration energy dissipation during Phase III is given by

$$\Delta E_{vis,III} = \frac{3}{8}\pi\mu_l d_p^2 v_{p,II}\left(\ln\frac{\delta_l}{\epsilon_{pr}} - \ln\frac{\delta_{pr}}{\epsilon_{pr}}\right) \tag{16}$$

$$\Delta E_{acc,III} = \frac{1}{2}\rho_l \left[\pi d_p^3 \left(\frac{\delta_l}{d_p}\right)^2 \left(\frac{1}{2} - \frac{\delta_l}{3d_p}\right)\right] \left[v_{p,II}\left(\frac{d_p}{\delta_l} - 1\right)^{0.5}\right]^2 \tag{17}$$

Note that, in the viscous damping, the additional energy dissipated during the formation of the liquid bridge can be taken into account as reported by Ref. 20, where δ_{pr} is the breakage distance of the liquid bridge. The restitution coefficient during Phase III can be obtained as

$$e_{III} = \sqrt{\left(\frac{E_{II} - \Delta E_{vis,III} - \Delta E_{acc,III}}{E_{II}}\right)} \tag{18}$$

Phase IV: The energy lost during rupture of the liquid bridge in Phase IV was reported by Ref. 20 and reads

$$\Delta E_{b,IV} \approx \pi\gamma_l \sqrt{2V_b d_p} \tag{19}$$

where the liquid bridge volume generally equals to $V_b \approx d_p^3/16$. The restitution coefficient during Phase IV can be obtained as

$$e_{IV} = \sqrt{\left(\frac{E_{III} - \Delta E_{b,IV}}{E_{III}}\right)} \tag{20}$$

Phase V: The particle kinetic energy after the breakage of the liquid bridge can be calculated by estimating the volume of the liquid attached to the particle, which is approximated by determining the mass of the spherical cap as

$$m_{cap} = \pi d_p^3 \left(\frac{\delta_l}{d_p}\right)^2 \left(\frac{1}{2} - \frac{\delta_l}{3d_p}\right)^2 \rho_l \tag{21}$$

$$\Delta E_{am,V} = \frac{1}{2} m_{cap} v_{p,IV}^2 \tag{22}$$

$$e_V = \sqrt{\left(\frac{E_{IV} - \Delta E_{am,V}}{E_{IV}}\right)} \tag{23}$$

Finally, the overall effective wet restitution coefficient is given by the ratio of the particle velocity after Phase V and the initial impact velocity 0, that is, the product of the energy dissipated during the various phases as

$$e_{wet} = \frac{v_{p,V}}{v_{p,0}} = \prod_{i=I}^{V} e_i \tag{24}$$

Results and Discussions

This section contains the following elements: (1) experimental determination of the dry and wet restitution coefficient, (2) collection of the experimental datasets, (3) evaluation of the coefficients of the unified correlation and comparison with experimental data and data from a elastohydrodynamic model, and (4) evaluation of the energy analysis model and comparison with experiments, elastohydrodynamic model and unified correlation.

Determination of the restitution coefficient

Dry Restitution Coefficient. Collision experiments were carried out as described in section. Variation in the dry restitution coefficient with particle impact velocities were determined by adjusting the nozzle height in a vertical direction and capturing high-speed images. The implementation of the developed script and the experimental procedure were double-checked by comparing the results with literature data. The variation in the particle location with time is shown in Figure 4 and the corresponding restitution coefficient in Figure 5. The restitution coefficient decreases with impact velocity, which is in accordance with an earlier experimental study by Ref. 22. This is mainly due to the energy transfer from translational motion to rotational direction, which is attributed to the microscopic particle roughness.

Wet Restitution Coefficient. A similar experimental procedure was applied to determine the wet restitution coefficient for different physical properties by changing the composition of a water/glycerol solution (as shown in Table 2) and liquid layer thickness $\delta_l = 500$–1000 μm.

Figure 6 illustrates variation in the particle velocity (after release from the nozzle) with time, along with high-speed

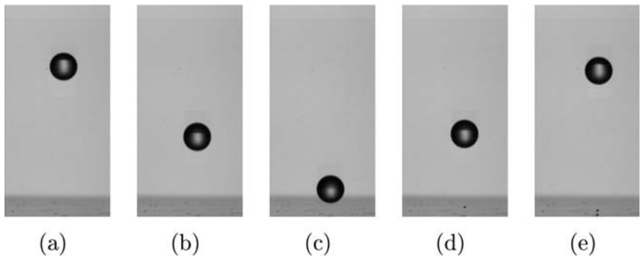


Figure 4. High-speed images of the impact of a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a dry flat glass plate.

images captured at 4000 frames per second during the collision of the glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a flat wet plate covered with water of 1 mm thickness (0.4 times particle diameter). First, the particle accelerates downward on collision with the wet plate. After the collision, the particle changes its direction and accelerates upward. As the particle approaches the liquid layer (at $t = 16$ ms), it pushes the air (present in between the particle and liquid layer) in the lateral direction. This leads to the formation of a circular wave pattern on the liquid surface with a liquid splash. The dynamics of the liquid splash highly depend on the liquid layer thickness, particle impact velocity, and viscosity. As the particle penetrates the liquid layer, it pushes the surrounding liquid away and rebounds (at $t = 17.8$ ms) with formation of a liquid bridge between the particle and the liquid surface (at $t = 21.3$ ms). The liquid bridge continuously deforms and elongates till it finally ruptures, which is often associated with the formation of satellite droplets. In addition, some part of the formed liquid bridge gets attached to the particle (i.e., added mass shown at $t = 37.5$ ms). As time progresses, the liquid bridge becomes asymmetric after particle detachment, which may be because the added liquid mass tends to unbalance the particle motion. So, during these series of events a considerable amount of energy has been dissipated, attributing to a lower overall restitution coefficient compared to the dry impact.

The variation in the wet restitution coefficient for other liquid properties and liquid layer thicknesses were determined and the obtained results were used for the development and validation of the correlation. Figure 7 illustrates the variation in the wet restitution coefficient with liquid layer thickness and viscosity. For water at a liquid layer thickness $\delta_l = 1000$ μm, the wet restitution coefficient increases with impact velocity and then remains constant. Similar trends were observed for a higher liquid layer thickness but with lower restitution coefficients, as shown in Figure 7a. As the liquid layer thickness increases, e_{wet} decreases, due to the pronounced viscous energy dissipation in the liquid layer. Additionally, an effect of the liquid viscosity and surface tension on the wet restitution coefficient was determined by changing the glycerol concentration (Figure 7b). For a constant liquid layer thickness with increased liquid viscosity, e_{wet} decreases.

The dynamics involved during formation and breakage of the liquid bridge and satellite droplets depend on the physical properties of the liquid (viscosity), liquid layer thickness, and particle velocity. During particle rebound from the liquid layer (at high particle velocities or low liquid layer thickness) a certain amount of the liquid attaches to the particle with the formation and breakage of a liquid bridge and satellite droplets. Figures 8 and 9 show the snapshots for the liquid bridge just before rupture and the formed satellite droplets for liquid layer thicknesses of $\delta_l = 500$–1000 μm. For a liquid layer ($\delta_l = 1000$ μm) with a low viscosity, a thick neck liquid bridge with a wider base was observed as compared to more viscous liquid (as shown in Figure 8a,c). Additionally, for low viscosity liquids, the liquid bridge length is considerably shorter and ruptures quicker. For a viscous liquid more of liquid is attached to the particle, which may be due to the high viscous force. This added mass significantly lowers the restitution coefficient.

For low viscous liquid and low layer thickness (at a same impact velocity), a very thin liquid bridge (narrow base) was observed with a high number of tiny satellite droplets (Figure 9a,b).

Multiple Wet Collisions. In gas-solid contactors with liquid injection often multiple successive particle–particle–liquid interactions take place. Figure 10 shows the change in the particle velocity with time, along with snapshots captured of a wet collision of the glass particle on a wet flat plate covered with glycerol solution (25% water + 75% glycerol [w/w]) of $\delta_l = 1000$ μm.

After particle is released from the nozzle, the particle velocity slightly increases and then remains constant to the terminal settling velocity. Once the particle hits the wall, its velocity inverts and leaves the liquid layer. Similar behavior was observed for the second collision, however, in this case, the particle does not rebound because it sticks to the liquid layer. This kind of investigation can be useful to study time required for the particle sticking in agglomeration application. Here, it is noted that, the wet restitution coefficient significantly decreases with the number of impacts (i.e., with the reduction in impact velocity). Also, the particle rebound trajectory is not parabolic (as compared with dry impact reported by Ref. 20), not only due to the constant gravitational force but also due to the drag force and the added liquid mass (Figure 10, for $t = 78.5$ ms).

Collection of datasets

A wide range of experimental data used for evaluation of the wet restitution coefficient from open literature is summarized in Table 3. Note that the experimental data is

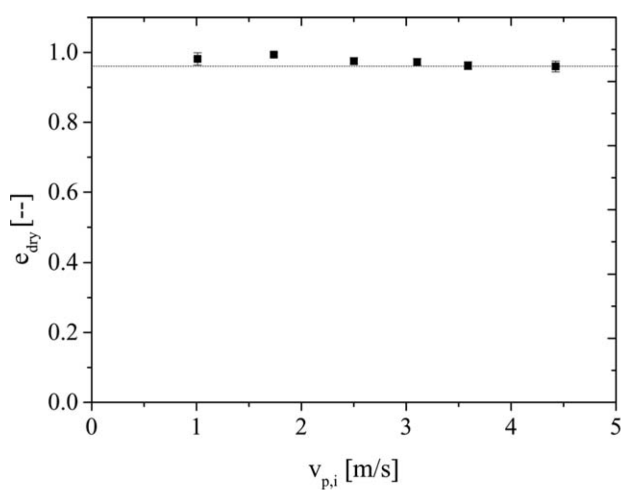


Figure 5. Variation in the dry restitution coefficient for the impact of a glass particle on a dry flat plate for different impact velocities.

Table 2. Physical Properties of the Systems Studied in This Work

Physical properties[a]	Water	50% Water + 50% glycerol	25% Water + 75% glycerol	Unit
Liquid viscosity	1×10^{-3}	6.35×10^{-3}	9.59×10^{-3}	kg/ms
Liquid density	998	1125	1198	kg/m^3
Liquid surface tension	72×10^{-3}	70.8×10^{-3}	69.1×10^{-3}	N/m

[a]The physical properties of water and glycerol solution were measured using a viscosimeter, a specific gravity bottle and a surface tension meter.

collected from studies comprising a similar experimental procedure, that is, impinging a spherical particle on a stationary plate covered with a thin liquid layer. Gollwitzer et al.[20] investigated the effect of impact velocity, liquid viscosity, and layer thickness on wet restitution coefficient along with viscous energy dissipation rate for glass particles ($d_p = 2.8$–10 mm and $\rho_p = 2850$ kg/m^3) by changing the physical properties of the liquid (water, M5, and M50 oil) and impact velocities. Further, our experiments also involve a similar study for glass particles ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) by changing the physical properties of the liquid and impact velocities.

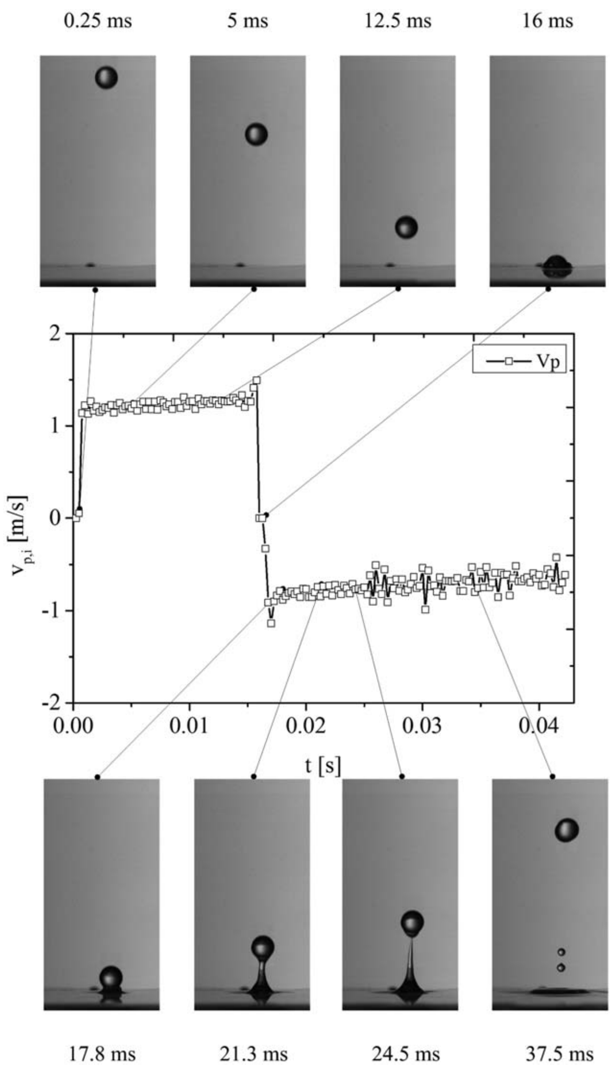


Figure 6. Variation in the particle velocity along with the snapshots captured using a high speed camera at 4000 fps during the collision of a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a flat wet plate covered with water of $\delta_l = 1000$ μm.

A unified correlation

For a unified correlation with coefficients, a linear regression was carried in a SPSS software and the obtained coefficients for each dimensionless groups are substituted in Eq. 5 as

$$\Delta E_{wet} = 0.959\left(\frac{d_p}{\delta_l}\right)^{-0.08} We_p^{0.19} Re_l^{-0.025} St^{-0.17} \quad (25)$$

The accuracy of the developed correlation was analyzed by plotting a parity plot for a wide range of experimental

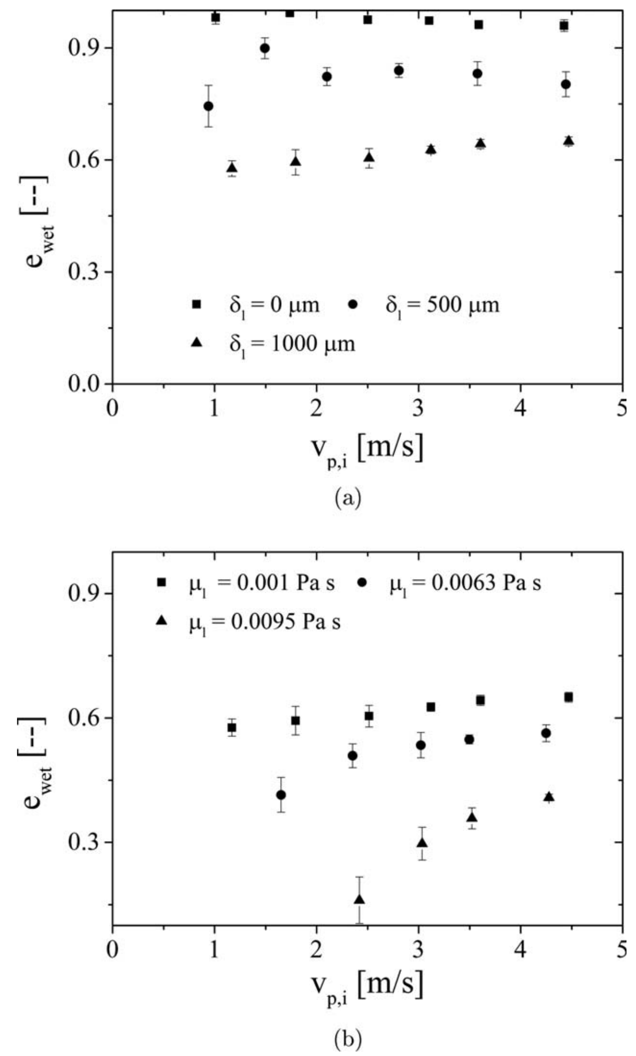


Figure 7. Variation in the wet restitution coefficient for a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a wet flat plate for different impact velocities and (a) different liquid layer thicknesses ($\delta_l = 1000$ μm) and (b) different viscosities ($\mu_l = 0001$–00095 Pa s) at $\delta_l = 1000$ μm.

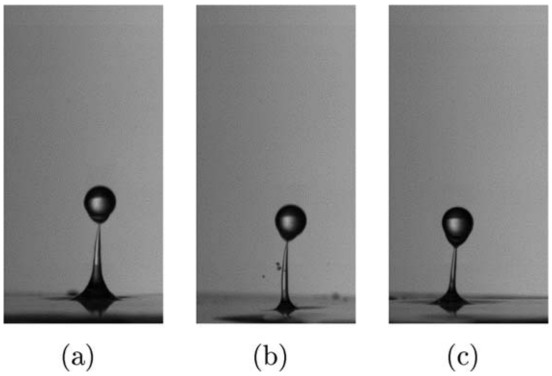


Figure 8. Images showing the liquid bridge just prior to rupture during a wet collision of a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a wet flat plate covered by a liquid (a) water of $\delta_l = 1000$ μm, (b) water of $\delta_l = 500$ μm, and (c) glycerol (25% water +75% glycerol [w/w]) of $\delta_l = 1000$ μm.

data as shown in Figure 11, comparing the experimental and correlation results. The estimation seems to work better for the high restitution coefficients (high impact velocities, low viscosities). This may be due to the unavailability of the precise experimental data at lower values of the restitution coefficient. This can be further improved using new data.

The efficacy of the developed correlation can be evaluated by comparing the correlation results with independent experiments, and the elastohydrodynamic model by Davis et al.[9] for smooth surfaces. The magnitude of the particle rebound defined in terms of the wet restitution coefficient and reduced by a factor e_{dry} as

$$e_{wet} = e_{dry}\left(1 - \frac{St_c}{St}\right) St_c < St \tag{26}$$

Equation 26 is valid for $St_c < St$. No particle rebound was observed at $St < St_c$ due to a significant loss in the particle kinetic energy in the liquid (viscous dissipation). The limiting value of a critical Stokes number (St_c), below which the particle will not rebound is given by

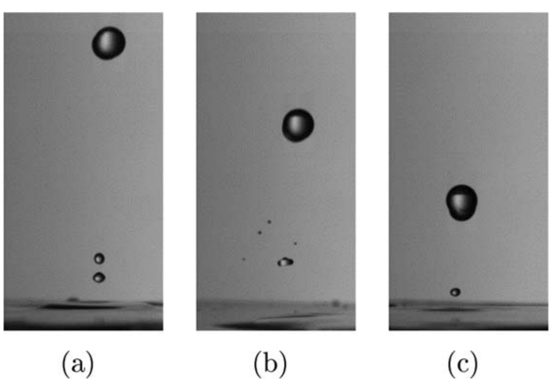


Figure 9. Images showing the formed satellite droplets after rupture of the liquid bridge during wet collision of a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a wet flat plate covered by a liquid (a) water of $\delta_l = 1000$ μm, (b) water of $\delta_l = 500$ μm, and (c) glycerol (25% water +75% glycerol [w/w]) of $\delta_l = 1000$ μm.

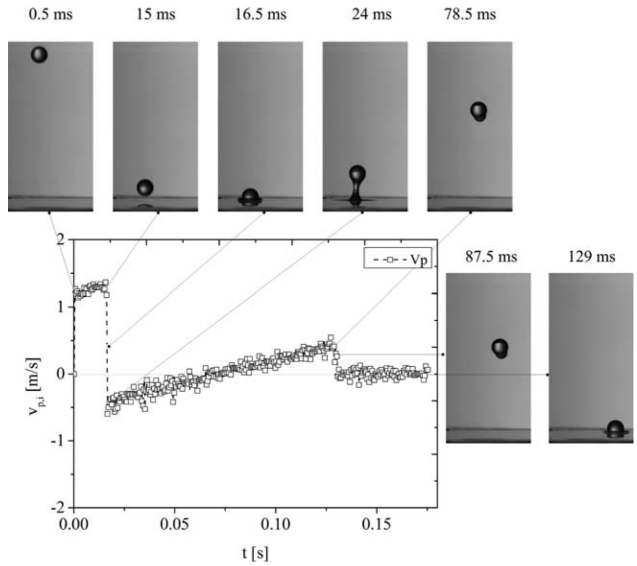


Figure 10. Variation in the particle velocity along with snapshots capture of a wet collision of a glass particle ($d_p = 2.5$ mm and $\rho_p = 2526$ kg/m^3) on a wet flat plate covered with glycerol solution (25% water + 75% glycerol [w/w]) of $\delta_l = 1000$ μm.

$$St_c = 0.4\ln\left(\frac{1}{\epsilon'}\right) - 0.20 \tag{27}$$

where ϵ' is the dimensionless elasticity parameter calculated as

$$\epsilon' = \left(\frac{4\theta v_{p,0} r_p^{3/2}}{z_0^{5/2}}\right) \tag{28}$$

The effective Young's modulus (Y_{eff}) depends on the Poisson's ratio (υ) and Young's modulus (Y) for Particle (1) and Plate (2) as

$$Y_{eff} = \frac{(1-v_1^2)}{\pi Y_1} + \frac{(1-v_2^2)}{\pi Y_2} \tag{29}$$

Generally, the particle has to penetrate to a certain depth in the liquid layer before the viscous forces become significant. Reference 9 estimated this distance as $z_0 = 2/3\delta_l$. The comparison of the correlation results with experimental results (our own experiments and from Ref. 20), and elastohydrodynamic model are shown in Figure 12. From this, it can be observed that the proposed correlation can produce more accurate agreement with experiment than the elastohydrodynamic model.

Table 3. Range of Dominant Parameters Used for Evaluation of the Wet Restitution Coefficient from Open Literature

Parameters	Range	Dimension
Particle diameter	$2.5 < d_p < 10$	mm
Particle density	$2526 < \rho_p < 2580$	kg/m^3
Stokes number	$26 < St < 4288$	–
Particle Reynolds number	$87 < Re_p < 15,250$	–
Weber number	$1.1 < We_p < 1,126$	–
Liquid layer thickness	$20 < \delta_l < 1000$	μm
Liquid viscosity	$1\times10^{-3} < \mu_l < 48\times10^{-3}$	Pa s
Liquid surface tension	$19\times10^{-3} < \gamma_l < 78\times10^{-3}$	kg/s^2
Liquid density	$881 < \rho_l < 998$	kg/m^3

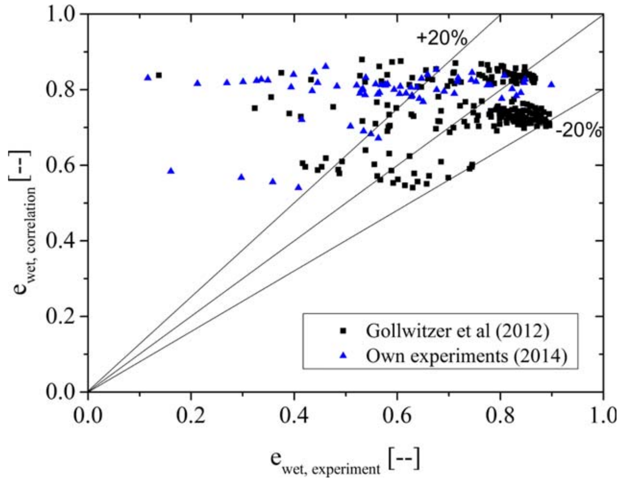


Figure 11. Comparison of the wet restitution coefficient obtained using correlation (Eq. 25) and experimental data with average relative error equal to 0.26; the solid line indicate ± 20% deviation.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

In the elastohydrodynamic model, the wet energy loss is mainly characterized by viscous forces (neglecting, e.g., the capillary forces). With this assumption, the energy loss can be characterized by the Stokes number (ratio of particle inertia and liquid viscosity). As such, this model is applicable to study the wet particle collision at low liquid Reynolds numbers $Re_l \ll 1$ and a low instantaneous distance between the nose of the undeformed particle and substrate. It essentially indicates that this model can be applied either at high viscous forces or low particle inertia forces. So at a high liquid Reynolds number, the total energy dissipated in the liquid layer due to viscous damping may not be pronounced. In addition, no particle rebound was observed at $St < St_c$ due to a significant loss in the particle kinetic energy in the liquid

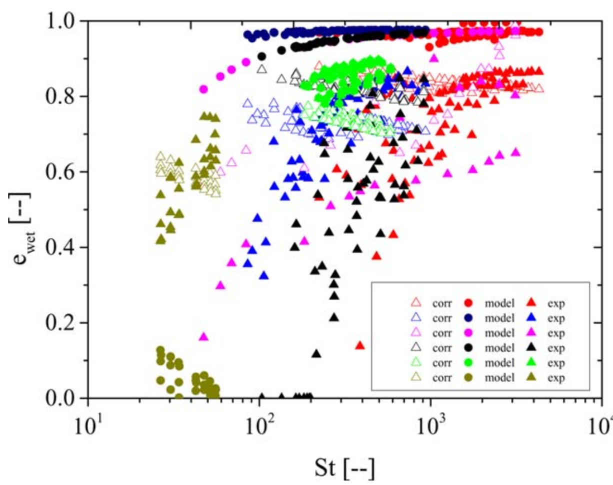


Figure 12. Comparison of the correlation results with: experiments by Ref. 20, our experiments, and an elastohydrodynamic model proposed by Ref. 9.

(Red, blue, green, and light green: experimental data from Ref. 20; pink and black: our experiments.) [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

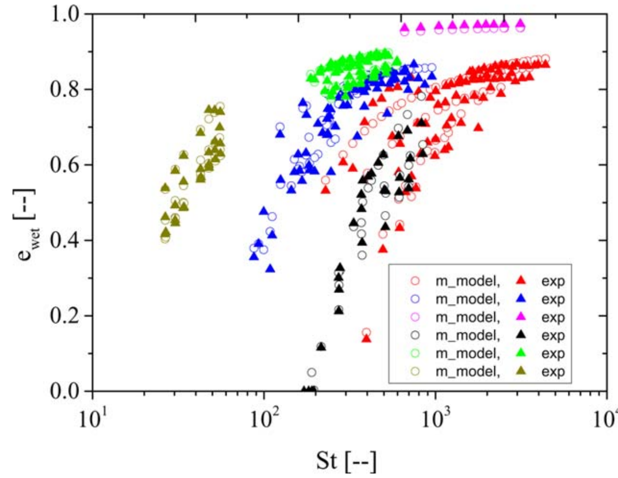


Figure 13. Comparison of the modified elastohydrodynamic model (with $e_\infty = 0.75–0.966$ and $St_{c,\infty} = 45–2278$) results with our experiments performed by impacting glass ($d_p = 2.5$ mm) particle on a glass plate covered with water ($\delta_l = 1000$ μm).

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

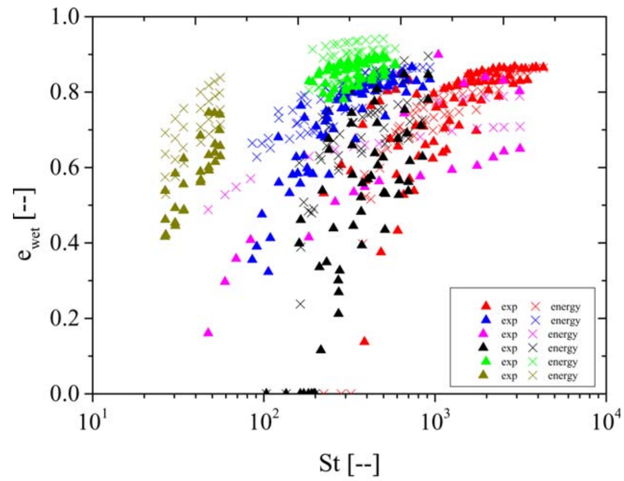
(viscous dissipation). An evaluation of the model predicts that the critical Stokes number increases with the elasticity parameter and for a constant Stokes numbers, the wet restitution coefficient decreases with increase in St_c. However, the wet restitution coefficient depends stronger on the Stokes number than on the elasticity parameter (or critical Stokes numbers). The limiting value of St_c, below which the particle will not rebound was evaluated either empirically through experiments or theoretically by smooth surfaces (for both particle and plate) and considering physical properties of both the liquid and the particles. But in reality, the colliding particles often possess microscopic roughness. Furthermore, for a given liquid at a constant liquid layer thickness, the particle rebound is not necessarily reduced by factor e_{dry}, but instead by a lower value e_∞ (a saturated value of wet restitution coefficient at a high Stokes numbers). For all datasets (shown in Figure 12) at lower Stokes numbers, e_{wet} increases rapidly and remains constant at high Stokes numbers. The modified expression for the wet restitution coefficient becomes

$$e_{wet} = e_\infty \left(1 - \frac{St_{c,\infty}}{St}\right) St > St_{c,\infty} \tag{30}$$

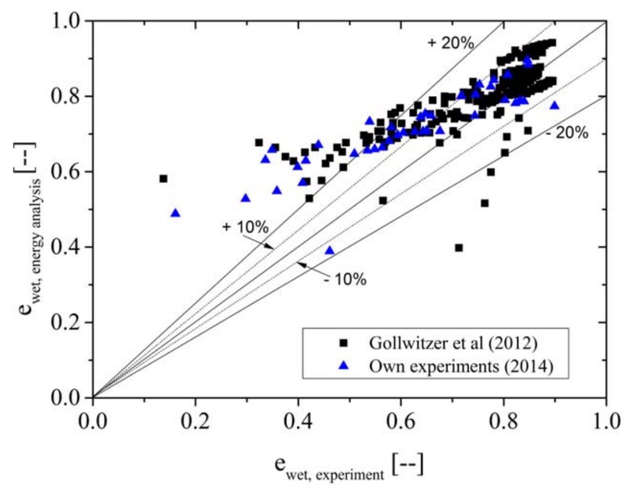
Reference 20 reported that the e_∞, initially increases and after reaching a maximum value start decreasing with particle diameter and Ref. 23 reported that e_∞ decrease with liquid layer thickness. Therefore, the model proposed by Ref. 9 can be modified for higher liquid Reynolds number by estimating saturated critical Stokes number ($St_{c,\infty}$) and saturated wet restitution coefficient. Figure 13 illustrates the comparison of the experimental with modified elastohydrodynamic model, indicate $e_\infty = 0.75–0.976$ with saturated critical Stokes number, $St_{c,\infty} = 47–2278$. However, this approach is not very practical since the values of e_∞ and $St_{c,\infty}$ are not unique.

Energy analysis

The efficacy of the developed energy approach can be analyzed by comparing the results with experiments as



(a)



(b)

Figure 14. Comparison of the energy analysis results with experiments (a) along with parity plot (b), with average relative error equal to 0.152; solid and dotted lines indicate ±20 and ±10% deviation. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

shown in Figure 14. It is found that the energy analysis shows a similar agreement as compared to the empirical correlation, despite the fact that no explicit fitting parameters were used in the analysis. It is noted that, the proposed energy approach can be used for particles possessing a small microscopic roughness, which was not considered in the elastohydrodynamic model and the correlation proposed earlier in this work. Initially, the comparison of the results were carried by assuming a particle roughness of 5 μm. As the particle roughness decrease, e_{wet} also decreases. So the accurate evaluation of the wet restitution coefficient for the particles with microscopic imperfections can also be accounted for in the energy analysis.

Conclusions

Dynamics of the gas-solid contactors with liquid injection highly depend on the viscous energy dissipation rate during effective particle interactions and are expressed in terms of a wet restitution coefficient (e_{wet}). In this study, two approaches were used to develop an expression for e_{wet}: through dimensional analysis and through an energy analysis. The correlation resulting from the dimensional analysis was developed using the Buckingham π theorem and regression analysis, by identifying the dominant physical properties of the solid and liquid, for 252 data points. The dominant physical properties of solid and liquid, particle impact velocity, and liquid layer thickness were grouped into well-known dimensionless numbers viz. Reynolds, Weber, and Stokes. Additionally, a more rigorous approach, for example, an energy analysis was taken by calculating the particle kinetic energy during five distinct phases observed during the impact, that is, dipping, solid contact, undipping, formation, and breakage of the liquid bridge and added mass. The efficacy of the developed approaches were analyzed by comparing the results with elastohydrodynamic model[9] and experiments, performed by impacting a spherical particle on a stationary glass plate covered with a thin liquid layer of water and glycerol solution. It was found that the dimensional and energy analysis can comparatively well describe the experimental data as compared with the elastohydrodynamic model. Thus, the proposed approaches can be used as an input expressions in particle-based modelling approaches involving liquid injection such as CFD-DEM, in which the particle collision behavior strongly depends on the accuracy of the wet restitution coefficient. Additionally, a modified elastohydrodynamic model was used to determine the e_{wet} by calculating the saturated wet restitution coefficient and critical Stokes number.

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Notation

Roman symbols

Ca = capillary number
D = depth of the plate, cm
d_p = particle diameter, m
E_a = total energy after impact, J
E_b = total energy before impact, J
e_{wet} = wet restitution coefficient
e_{dry} = dry restitution coefficient
e_∞ = saturated restitution coefficient
H = height of the plate, cm
m_p = particle mass, kg
m_{cap} = mass of spherical cap, kg
Re_l = liquid Reynolds number
St = Stokes number
St_c = critical Stokes number
$St_{c,\infty}$ = saturated critical Stokes number
t = time, ms
v_b = volume of liquid bridge, m^3
$v_{p,0}$ = particle impact velocity, m/s
$v_{p,a}$ = particle velocity after impact, m/s
$v_{p,b}$ = particle velocity before impact, m/s
W = width of the plate, cm
We_p = Weber number

Y_{eff} = effective Young's modulus, N/m
Y = Young's modulus, N/m

Greek symbols

θ_i = impact angle, °
ρ_p = density of the particle, kg/m^3
ρ_l = density of the liquid, kg/m^3
δ_l = liquid layer thickness, m
δ_{pr} = liquid bridge breaking distance, m
μ_l = viscosity of the liquid, μm
γ_l = surface tension of the liquid, N/m
ΔE_{wet} = dimensionless energy loss
ΔE_{loss} = energy loss, J
π = dimensionless group
ε' = elasticity parameter
υ = Poisson's ratio

Abbreviations and subscripts

CFD = computational fluid dynamics
DEM = discrete element model

Literature Cited

1. Bruhns S, Werther J. An investigation of the mechanism of liquid injection into fluidized beds. *AIChE J*. 2005;51:766–775.
2. Raman CV. The photographic study of impact at minimal velocities. *Phys Rev*. 1918;12:442–447.
3. Uchiyama Y, Arakawa M, Okamoto C, Yasui M. Restitution coefficients and sticking velocities of a chondrule analogue colliding on a porous silica layer at impact velocities between 0.1 and 80 ms-1. *Icarus*. 2012;219:336–344.
4. Gorham DA, Kharaz, AH. The measurement of particle rebound characteristics. *Powder Technol*. 2000;112:193–202.
5. Mueller P, Antonyuk S, Stasiak M, Tomas J, Heinrich S. The normal and oblique impact of three types of wet granules. *Granular Matter*. 2011;13:455–463.
6. Mangwandi C, Cheong YS, Adams MJ, Hounslow MJ, Salman AD. The coefficient of restitution of different representative types of granules. *Chem Eng Sci*. 2007;62:437–450.
7. Marinack MC Jr, Musgrave RE, Higgs, CF III. Experimental investigations on the coefficient of restitution of single particles. *Tribol Trans*. 2013;56:572–580.
8. Joseph GG, Zenit R, Hunt ML, Roseneinkel AM. Particle wall collisions in a viscous fluid. *J Fluid Mech*. 2001;433:329–346.
9. Davis RH, Ragerd DA, Good BT. Elastohydrodynamic rebound of spheres from coated surfaces. *J Fluid Mech*. 2002;468:107–119.
10. Gondret P, Lance M, Petit L. Bouncing motion of spherical particles in fluids. *Phys Rev E*. 2002;14:643–652.
11. Fu JS, Cheong YS, Reynolds GK, Adams MJ, Salman AD, Hounslow MJ. An experimental study of the variability in the properties and quality of wet granules. *Powder Technol*. 2004;140:209–216.
12. Fu J, Adams MJ, Reynolds GK, Salman AD, Hounslow MJ. Impact deformation and rebound of wet granules. *Powder Technol*. 2004; 140:248–257.
13. Kantak AA, Davis RH. Collisions of spheres with wet and dry porous layers on a solid wall. *Chem Eng Sci*. 2006;61:417–427.
14. Stocchino A, Guala M. Particle-wall collision in shear thinning fluids. *Exp Fluids*. 2005;38:476–484.
15. Antonyuk S, Heinrich S, Deen NG, Kuipers JAM. Influence of liquid layers on energy absorption during particle impact. *Particuology*. 2009;7:245–259.
16. Subbarao KKS. Particulate fouling of dry and liquid coated surfaces, PhD thesis, Eindhoven University of Technology, The Netherlands, 2012.
17. Donahue CM, Davis RH, Kantak AA, Hrenya CM. Mechanisms for agglomeration and deagglomeration following oblique collisions of wet particles. *Phys Rev E*. 2012;86:021303.
18. Donahue CM, Brewer WM, Davis RH, Hrenya CM. Agglomeration and de-agglomeration of rotating wet doublets. *J Fluid Mech*. 2012; 708:128–148.
19. Jain D, Deen NG, Kuipers JAM, Antonyuk S, Heinrich S Direct numerical simulation of particle impact on thin liquid flim using a combined volume of fluid and immersed boundary method. *Chem Eng Sci*. 2012;69:530–540.
20. Gollwitzer F, Rehberg I, Kruelle CA, Huang K Coefficient of restitution for wet particles. *Phys Rev E*. 2012;86:011303–011308.
21. Ennis BJ, Tardos G, Pfeffer R. A microlevel-based characterization of granulation phenomena. *Powder Technol*. 1991;65:251–272.
22. Montaine M, Heckel M, Kruelle CA, Schwager T, Pöschel T Coefficient of restitution as a fluctuating quantity. *Phys Rev E*. 2011;84: 041306.
23. Müller T, Gollwitzer F, Krulle CA, Rehberg I, Huang K. Scaling of the normal coefficient of restitution for wet impacts. *AIP Conf Proc*. 2013;1542:787–790.

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